

# Quantum Chromodynamics with massive gluons

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## Abstract

It is shown that the Lagrangian of Quantum Chromodynamics can be modified by the adding gluon masses. On mass-shell renormalizability of the resulting theory is discussed.

The discovery [1] of asymptotic freedom in Quantum Chromodynamics (QCD) has lead to the establishment of QCD as the theory of strong interactions. The gauge bosons of the theory, the gluons, are considered to be massless to have gauge invariance and correspondingly renormalizability. In the present paper it is shown that the QCD Lagrangian should be modified by the adding gluon masses to ensure that QCD does not contradict to experiments. On mass-shell renormalizability of the resulting theory is discussed.

The Lagrangian of QCD is

$$L_{QCD} = -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + i\bar{\psi}_f \gamma_\mu D_\mu \psi_f - m_f \bar{\psi}_f \psi_f \quad (1)$$

$$-\frac{1}{\xi}(\partial^\mu A_\mu^a)^2 + \partial^\mu \bar{c}^a (\partial_\mu c^a - g f^{abc} c^b A_\mu^c) + \text{counterterms},$$

where  $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$  is the gluon field strength tensor,  $D_\mu = \partial_\mu - ig A_\mu^a T^a$  is the covariant derivative. The quark fields  $\psi_f$  transform as the fundamental representation of the colour group  $SU(3)$ ,  $f = u, d, s, c, b, t$  is the flavour index. The gluons  $A_\mu^a$  transform as the adjoint representation of this group.  $c^a$  are the ghost fields,  $\xi$  is the gauge parameter of the usually chosen general covariant gauge,  $f^{abc}$  are the structure constants of the group,  $T^a$  are the generators of the fundamental representation.  $g = g(\mu)$  is the renormalized strong coupling constant,  $g^2/(16\pi^2) \equiv a_s$ ,  $m_f = m_f(\mu)$  is the Lagrangian (renormalized) mass of a quark with a flavor  $f$ , and  $\mu$  is the renormalization point. The summations over repeated indexes are assumed.

Let us consider the vacuum polarization function  $\Pi(q^2)$

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu) \Pi(q^2) = i \int dx e^{iqx} \langle 0 | T j_\mu(x) j_\nu(0) | 0 \rangle. \quad (2)$$

where  $j_\mu = \sum_f q_f \bar{\psi}_f \gamma_\mu \psi_f$  is the electromagnetic quark current and  $q_f = 2/3, -1/3, \dots$  is the electromagnetic charge of the quark with a flavor  $f$ .

According to general principles of local quantum field theory the function  $\Pi(q^2)$  satisfies the Källén-Lehmann [2] spectral representation

$$\Pi(q^2) = \frac{1}{12\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s - q^2 - i0}, \quad (3)$$

where the ratio  $R(s) = \sigma_{tot}(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  is the normalized total cross-section of electron-positron annihilation into hadrons,  $m_\pi$  is a pion mass.

The Källen-Lehmann representation determines the analytic properties of  $\Pi(q^2)$  which should be an analytic function in the complex  $q^2$ -plane with the cut starting from the first physical threshold, i.e. as it is dictated by experiments from the two-pion threshold  $q^2 = 4m_\pi^2$ . In particular, one gets for the discontinuity of  $\Pi(q^2)$  on the cut

$$\Delta\Pi(q^2) \equiv \Pi(q^2 + i0) - \Pi(q^2 - i0) = \begin{cases} i R(q^2)/(6\pi) & \text{at } s > 4m_\pi^2 \\ 0 & \text{at } s < 4m_\pi^2. \end{cases} \quad (4)$$

Perturbative QCD produces the following expression for the discontinuity

$$\Delta\Pi(q^2)_{pQCD} = \theta(q^2) \rho_{gluon}(q^2) + \theta(q^2 - 4M_u^2) \rho_{quark}(q^2). \quad (5)$$

The gluon spectral density  $\rho_{gluon}(q^2)$  contributes for  $q^2 > 0$  as it is indicated by the theta-function  $\theta(q^2)$ . This is the known zero threshold. It arises from those absorptive parts of Feynman diagrams of  $\Pi(q^2)$  which are produced by purely gluonic cuts of the diagrams (i.e. by the Cutcosky cuts which cross only gluon propagators of diagrams). As it is well known such diagrams appear for the first time at the four-loop level in the order  $a_s^3$  (corresponding cuts cross 3 gluon propagators).

The quark spectral density  $\rho_{quark}(q^2)$  arises from the quark cuts of the diagrams (i.e. from the cuts which cross two or more quark propagators of the diagrams). It contributes for  $q^2 > 4M_u^2$  where  $M_u$  is the perturbative pole mass of the lightest  $u$ -quark, defined as the pole of the quark propagator within perturbation theory. A perturbative quark pole mass

$$M_f = m_f(\mu) + O(a_s) \quad (6)$$

appears after summation of perturbative corrections to a quark propagator. It is a renormalization group invariant quantity, i.e. independent on the renormalization point  $\mu$  and on the choice of the subtraction scheme. In this sense it behaves as a physical object and that is why it is natural to use this definition of a quark mass to parametrize the theory.

We will not discuss here the important by themselves questions of convergence or divergence of corresponding perturbative QCD series at low or at high energies. Here we will just accept that our conventional perturbation theory is adequate to the exact solution of the theory, i.e. it correctly reproduces the perturbative expansion of the exact solution.

Hence one gets within QCD that  $\Delta\Pi(q^2)$  is non-zero in the energy interval  $0 < q^2 < 4m_\pi^2$  since the perturbative contribution  $\Delta\Pi(q^2)_{pQCD}$  is non-zero

in this interval. There are of course also non-perturbative contributions, i.e. contributions of the type of  $e^{-1/a_s}$  which are invisible in the perturbative expansion at  $a_s = 0_+$

$$e^{-1/a_s} = 0 \cdot a_s + 0 \cdot a_s^2 + \dots$$

But non-perturbative contributions can not exactly cancel the perturbative contribution in the continuous interval  $0 < q^2 < 4m_\pi^2$  because of the different dependence on  $a_s$ . To get that  $\Delta\Pi(q^2) = 0$  at  $0 < q^2 < 4m_\pi^2$  in agreement with experiments one should move perturbative gluon and quark thresholds above  $q^2 = 4m_\pi^2$ . That is why we should introduce the non-zero Lagrangian gluon masses.

Thus one obtains the following restrictions on the (perturbative pole) masses of gluons and quarks

$$(3M_{gl})^2 > 4m_\pi^2, \quad (7)$$

$$4M_u^2 > 4m_\pi^2.$$

Although the restriction on  $M_u$  seems to be quite strong for the lightest u-quark it is not excluded from the first principles.

To construct QCD with massive gluons we will follow the approach of [3]. Presently this is the only known way to get (on mass-shell) renormalizable theory of massive gluons without color scalars (color scalars are rejected by experiments). Within this approach one starts from a renormalizable theory with scalar fields using the Englert-Brout-Higgs mechanism of spontaneous symmetry breaking [4] and after transition to the unitary gauge removes remaining massive scalar fields. Thus we add to the massless QCD Lagrangian (1) the scalar part to begin with the following general Lagrangian

$$L_{QCD+scalars} = -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + i\bar{\psi}_f \gamma_\mu D_\mu \psi_f - m_f \bar{\psi}_f \psi_f + \quad (8)$$

$$\begin{aligned} & (D_\mu \Phi)^\dagger D_\mu \Phi + (D_\mu \Sigma)^\dagger D_\mu \Sigma - \lambda_1 \left( \Phi^\dagger \Phi - v_1^2 \right)^2 - \lambda_2 \left( \Sigma^\dagger \Sigma - v_2^2 \right)^2 \\ & - \lambda_3 \left( \Phi^\dagger \Phi + \Sigma^\dagger \Sigma - v_1^2 - v_2^2 \right)^2 - \lambda_4 \left( \Phi^\dagger \Sigma \right) \left( \Sigma^\dagger \Phi \right) \\ & + L_{gf} + L_{gc} + \text{counterterms}, \end{aligned}$$

where we introduced two triplets  $\Phi(x)$  and  $\Sigma(x)$  of complex scalar fields in the fundamental representation of the  $SU(3)$  color group to get all gluon massive.

$L_{gf}$  is the gauge fixing part of the Lagrangian in some chosen gauge and  $L_{gc}$  is the corresponding gauge compensating part with the Faddeev-Popov ghost fields.

We can choose the following shifts of scalar fields by the quantities  $v_1$  and  $v_2$  to generate masses of all eight gluons

$$\Phi(x) = \begin{pmatrix} \phi_1(x) + i\phi_2(x) + v_1 \\ \phi_3(x) + i\phi_4(x) \\ \phi_5(x) + i\phi_6(x) \end{pmatrix}, \quad \Sigma(x) = \begin{pmatrix} \sigma_1(x) + i\sigma_2(x) \\ \sigma_3(x) + i\sigma_4(x) + v_2 \\ \sigma_5(x) + i\sigma_6(x) \end{pmatrix}. \quad (9)$$

Choosing for simplicity  $v_1 = v_2 \equiv v$  one obtains the following massive terms for gluons in the Lagrangian

$$L_M = M^2 \left[ (A^1)^2 + (A^2)^2 + (A^3)^2 + \frac{1}{2}(A^4)^2 + \frac{1}{2}(A^5)^2 + \frac{1}{2}(A^6)^2 + \frac{1}{2}(A^7)^2 + \frac{1}{3}(A^8)^2 \right], \quad (10)$$

where  $M^2 \equiv g^2 v^2$  is the gluon mass parameter of the theory.

After the chosen shifts the following four combinations of scalar fields

$$\phi_1 + \frac{\lambda_3}{\lambda_1 + \lambda_3} \sigma_3, \quad \sigma_3, \quad \sigma_1 + \phi_3, \quad \sigma_2 - \phi_4 \quad (11)$$

become massive Higgs particles.

The following eight combinations

$$\sigma_1 - \phi_3, \quad \phi_4 + \sigma_2, \quad \phi_2 - \sigma_4, \quad \phi_2 + \sigma_4, \quad \phi_5, \quad \phi_6, \quad \sigma_5, \quad \sigma_6 \quad (12)$$

become massless Goldstone ghosts.

Now one can make transition to the unitary gauge. All ghost fields as usual disappear from the Lagrangian. Following the approach of [3] one can remove in the unitary gauge all Higgs fields from the Lagrangian preserving on mass-shell renormalizability of the theory. The Lagrangian of the resulting theory is

$$L_{massive \ QCD} = L_M - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + i\bar{\psi}_f \gamma_\mu D_\mu \psi_f - m_f \bar{\psi}_f \psi_f + \text{counterterms}, \quad (13)$$

where  $L_M$  is given in eq.(10).

Let us note that on mass-shell renormalizability does not mean that one should consider quarks and gluons as free external particles. It means that in the  $SU(3) \times SU(2) \times U(1)$  theory the  $S$ -matrix elements with the physical external particles will be finite.

One can calculate the one-loop  $\beta$ -function in this theory to obtain for a massless renormalization scheme (i.e. a scheme where renormalization group functions do not depend on masses) the following result

$$\beta(a_s) = \mu^2 \frac{\partial a_s}{\partial \mu^2} = \sum_{i \geq 0} \beta_i a_s^{i+2}, \quad (14)$$

$$\beta_0 = -\frac{32}{3} + \frac{2}{3}n_f,$$

here  $n_f$  is the number of active quark flavors.

Thus asymptotic freedom remains valid in the considered theory with massive gluons.

The author is grateful to collaborators of the Theory division of INR for helpful discussions. The work is supported in part by the grant for the Leading Scientific Schools NS-5590.2012.2 and by Federal Program 'Researches and developments of priority directions of science and technology in Russia' under contract No. 16.518.11.7072.

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